Diffusion-limited reaction for the one-dimensional trap system

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We have previously discussed the one-dimensional multitrap system of finite range and found somewhat unexpected result that the larger is the number of imperfect traps the higher is the transmission through them. We discuss in this work the effect of a small number of such traps arrayed along either a constant or a variable finite spatial section. It is shown that under specific conditions, to be described in the following, the remarked high transmission may be obtained for this case also. Thus, compared to the theoretical large number of traps case these results may be experimentally applied to real phenomena.

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I. INTRODUCTION

The problem of diffusion through traps [1–5] is generally discussed in the literature by refering to the single trap system. In Ref. [6] an infinite mutitrap system arranged over all space was discussed and the application of traps to long chain polymers was considered in Ref. [7]. In Ref. [8] the aspect of the density of one-dimensional traps in a finite spatial section was discussed and it was shown that the larger this density is the higher is the transmission of particles through this dense system of traps. In this work we refer, especially, to the small number of traps system and find the conditions under which the transmission through them is maximal. That is, assuming an ensemble of classical particles we look for the conditions that enable all or most of them to pass through the trap system. We apply in this work the transfer matrix method [9,10] used in Ref. [8] with respect to the multitrap system.

In Sec. II we introduce the one-dimensional N trap system, the relevant nomination and terminology and, especially, the appropriate transfer matrix formalism as in Ref. [8]. In Sec. III we pay special attention to a small section of the N trap system that includes m traps where m is a small number. We will find how the passage of classical particles through this subsystem is influenced by the relevant parameters of it. These parameters are the number m of traps, the total extent L of the subsystem, the ratio c of the total interval among its traps to the total width of them [8], the time at which the subsystem is observed and the degree k of its imperfection. Note that the ideal trap is characterized [2,3] by $k\rightarrow\infty$ in which case all the approaching particles are absorbed by it.

In Sec. IV we discuss the behavior of the transmission amplitude for some specific values of its parameters. This transmission amplitude is defined [8] as the ratio of the value of the imperfect trap coefficient of the transmitted particles at the last trap to its value at the first one. We show that this amplitude increases to unity for increasing c and for large values of the remarked variables L, and k. This amplitude, on the other hand, decreases with increasing t. Note that although one may expect the transmission to decrease for increasing k since this signifies, as remarked, a strengthening of the ideal character of the trap system which entails a large absorption of the passing particles, nevertheless the obtained results are exactly the opposite. We show that the deviation from the expected results is due to the initial and boundary-value conditions we employ for the imperfect trap system discussed here [see the set (3) in Ref. [8] and set (4) here], which are different from those of the ideal ones [see set (2) in Ref. [8] and set (3) here]; that is, the influence of the time factor that emerges from these conditions is much more large than that of k in such a way that it veils the expected influence of the latter especially during the initial time. We accompany our calculations with figures that exemplify the results obtained for representative values of L, k, m, and t.

II. TRANSFER MATRIX METHOD FOR THE IMPERFECT ONE-DIMENSIONAL TRAP SYSTEM

The problem we discuss here is the diffusion limited reaction in the presence of N traps where, compared to the case discussed in Ref. [8], N assumes small values. These traps are arrayed in an ordered one-dimensional structure along a spatial axis. We denote the total width of the traps by a and the total interval among them by b. Thus, for a total number of traps N the width of each is a/N and the interval between any two neighboring traps is $b/(N-1)$ since there are $(N-1)$ intervals among N traps. An important parameter related to this system is the ratio c of the total interval b to the total width a, that is, $c=b/a$. Thus, denoting the total length of the system $(a+b)$ by L one may express the parameters a and b as [8]

$$a=\frac{L}{(1+c)}, \quad b=\frac{Lc}{(1+c)}. \tag{1}$$

The relevant one-dimensional initial and boundary-value diffusion problem in the presence of N traps is

$$\begin{aligned} &\rho_t(x,t)=D\rho_{xx}(x,t), \quad 0<x\leqslant(a+b),\\ &\rho(x,0)=\rho_0+f(x), \quad 0<x\leqslant(a+b),\\ &\rho(x_i,t)=\frac{1}{k}\left.\frac{d\rho(x,t)}{dx}\right|_{x=x_i}, \quad t>0, \quad 1\leqslant i\leqslant 2N, \end{aligned} \tag{2}$$

where $\rho(x,t)$ denotes the density of the particles diffusing through the traps. $\rho_t(x,t)$ is the first-order partial derivative with respect to the time variable t and $\rho_x(x,t)$, $\rho_{xx}(x,t)$ are the first- and second-order partial derivatives with respect to the spatial variable x. D is the diffusion constant, which may be of two kinds, D_i and D_e, which are the diffusion constants inside and outside the traps, respectively. The second and third equations in set (2) are the initial and boundary-value conditions, respectively. The range of i in the third equation of set (2) is due to the fact that each trap has front and back faces. It has been shown [8] that the diffusion problem from set (2) may be separated into the following two problems:

$$\rho_t(x,t)=D\rho_{xx}(x,t), \quad 0<x\leqslant(a+b),$$
$$\rho(x,0)=f(x), \quad 0<x\leqslant(a+b), \tag{3}$$
$$\rho(x_i,t)=0, \quad t>0, \quad 1\leqslant i\leqslant 2N,$$

$$\rho_t(x,t)=D\rho_{xx}(x,t), \quad 0<x\leqslant(a+b),$$
$$\rho(x,0)=\rho_0, \quad 0<x\leqslant(a+b), \tag{4}$$
$$\rho(x_i,t)=\frac{1}{k}\frac{d\rho(x,t)}{dx}\bigg|_{x=x_i}, \quad t>0, \quad 1\leqslant i\leqslant 2N.$$

Set (3) is the one-dimensional initial and boundary-value diffusion problem in the presence of N ideal traps and set (4) is that in the presence of N imperfect traps. The general solution of set (2) is

$$\rho(x,t)=A\rho_1(x,t)+B\rho_2(x,t), \tag{5}$$

where $\rho_1(x,t)$ is the density of the ideal trap set (3) and $\rho_2(x,t)$ is that of the imperfect trap one (4). The appropriate $f(x)$ that satisfies the first and third equations of set (3) is $f(x)=\sin(\pi x/x_i)$. Thus, using the separation of variables method [12] one may write for $\rho_1(x,t)$ and $\rho_2(x,t)$ that satisfy the appropriate initial and boundary-value conditions:

$$\rho_1(x,t)=\sin\left(\frac{\pi x}{x_i}\right)\exp\left(-\frac{Dt\pi^2}{x_i^2}\right), \tag{6}$$

$$\rho_2(x,t)=\rho_0\left[\operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)+\exp(k^2Dt+kx)\times\operatorname{erfc}\left(k\sqrt{Dt}+\frac{x}{2\sqrt{Dt}}\right)\right]. \tag{7}$$

The $\operatorname{erf}(x)$ and $\operatorname{erfc}(x)$ are the error and complementary error functions, respectively, defined as [14]

$$\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du,$$

$$\operatorname{erfc}(x)=1-\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du.$$

We use in the following, as in Ref. [8], the transfer matrix method [9,10] an important element of which is the assumption that the density $\rho(x,t)$ and its partial derivative with respest to x changes continously along the section $L=(a+b)$. Thus, one may equate at each of the $2N$ faces of the N traps the $\rho(x,t)$ and $\rho_x(x,t)$ at one side of it to the corresponding quantities just at the other side as done in Ref. [8]. In such a way one may obtain $2N$ two-dimensional transfer matrices each of them relates the values of the coefficients A, B from Eq. (5) at one side of a face of a trap to the corresponding values of these coefficients at the other side of this face. We multiply together any two transfer matrices related to the two faces of the same trap so as to have one two-dimensional transfer matrix for each trap. Thus, denoting these matrices by T we may write the general transfer matrix equation for the one-dimensional N trap system that relates the coefficients A, B at the left face of the first trap to those at the right face of the last trap assuming that the diffusing particles enter the traps through their left faces [8].

$$\begin{pmatrix}A_{2N+1}\\ B_{2N+1}\end{pmatrix}=T(a+b)T\left(\frac{(N-1)(a+b)}{N}\right)\times T\left(\frac{(N-2)(a+b)}{N}\right)\cdots T\left(\frac{n(a+b)}{N}\right)\times T\left(\frac{(n-1)(a+b)}{N}\right)\cdots T\left(\frac{2(a+b)}{N}\right)\times T\left(\frac{a+b}{N}\right)\begin{pmatrix}A_1\\ B_1\end{pmatrix}. \tag{8}$$

Each T is, as remarked, a two-dimensional transfer matrix that relates the coefficients A and B of the ideal and imperfect trap density functions of the passing particles at one side of the relevant trap to those at the other side. The number of traps is N and by n we denote the general trap in this system. Note that each matrix T depends also [8] upon time t, the constant k [8], and two diffusion constants D_e and D_i. But, as seen in Ref. [8] [see also Eq. (8)], the matrices T differ from each other by only the values of x and share the same values of k, t, D_e, and D_i.

As seen in Ref. [8] the element T_{12} in each T is always zero and the other elements T_{11}, T_{21}, and T_{22} are given, for the value of $x=(a+b)/N$, by [8]

$$T_{11}\left(\frac{(a+b)}{N}\right)=\frac{\alpha\left(D_e,\frac{b}{N},t\right)\alpha\left(D_i,\frac{(a+b)}{N},t\right)}{\alpha\left(D_i,\frac{b}{N},t\right)\alpha\left(D_e,\frac{(a+b)}{N},t\right)}, \tag{9}$$

$$T_{21}\left(\frac{(a+b)}{N}\right)=\rho_0\left[\frac{\eta\left(D_i,\frac{(a+b)}{N},t\right)}{\eta\left(D_e,\frac{(a+b)}{N},t\right)}\left(\frac{\xi\left(D_e,\frac{b}{N},t\right)}{\eta\left(D_i,\frac{b}{N},t\right)}-\frac{\alpha\left(D_e,\frac{b}{N},t\right)\xi\left(D_i,\frac{b}{N},t\right)}{\alpha\left(D_i,\frac{b}{N},t\right)\eta\left(D_i,\frac{b}{N},t\right)}\right)\right]+\frac{\alpha\left(D_e,\frac{b}{N},t\right)}{\alpha\left(D_i,\frac{b}{N},t\right)}\left(\frac{\xi\left(D_i,\frac{(a+b)}{N},t\right)}{\eta\left(D_e,\frac{(a+b)}{N},t\right)}-\frac{\alpha\left(D_i,\frac{(a+b)}{N},t\right)\xi\left(D_e,\frac{(a+b)}{N},t\right)}{\alpha\left(D_e,\frac{(a+b)}{N},t\right)\eta\left(D_e,\frac{(a+b)}{N},t\right)}\right), \tag{10}$$

$$T_{22}\left(\frac{(a+b)}{N}\right)=\frac{\eta\left(D_e,\frac{b}{N},t\right)\eta\left(D_i,\frac{(a+b)}{N},t\right)}{\eta\left(D_i,\frac{b}{N},t\right)\eta\left(D_e,\frac{(a+b)}{N},t\right)}, \tag{11}$$

where α, ξ, and η in the former equations are given as (we write them for D_e)

$$\alpha(D_e,x,t)=\mathrm{erf}\left(\frac{x}{2\sqrt{D_e t}}\right)+\exp(k^2D_et+kx)\times\mathrm{erfc}\left(k\sqrt{D_et}+\frac{x}{2\sqrt{D_et}}\right), \tag{12}$$

$$\xi(D_e,x,t)=k\exp(k^2D_et+kx)\mathrm{erfc}\left(k\sqrt{D_et}+\frac{x}{2\sqrt{D_et}}\right), \tag{13}$$

$$\eta(D_e,x,t)=-\frac{\pi}{x}e^{-(\pi/x)^2D_et}. \tag{14}$$

III. THE ONE-DIMENSIONAL SMALL NUMBER OF TRAPS SYSTEM

It was shown in Ref. [8] for the multitrap system that the transmission coefficient, which was calculated as the ratio of the imperfect trap coefficient of the particles, after passing through the system to that before this passage tends to unity for the cases of (1) when the total length of the system $L=a+b$ grows, (2) when the total length L is constant and the ratio b/a of the total interval to the total width of the system increases. For these two cases the elements T_{21} and T_{22} tend to zero (as remarked, the value of the element T_{12} is always zero) and T_{11} tends to unity, which are the required conditions to obtain a unity value for the transmission coefficient. Moreover, it has been shown [8] that these specific values of the elements T_{21}, T_{22}, and T_{11} are, especially, obtained in the limit of $N\to\infty$. We want here to find if this kind of behavior may be discerned in small sections of the N system that contain small number of traps. We show in the following that the transmission coefficient may indeed assume, under certain conditions, a unity value for this case also. Thus, refering to a two-trap section in the N system we may write, using Eq. (8), the relevant matrix expression for it,

$$\begin{pmatrix}A_{2n+1}\\B_{2n+1}\end{pmatrix}=T\left(\frac{n(a+b)}{N}\right)T\left(\frac{(n-1)(a+b)}{N}\right)\begin{pmatrix}A_{2(n-2)+1}\\B_{2(n-2)+1}\end{pmatrix}$$

$$=\begin{bmatrix}T_{11}\left(\frac{n(a+b)}{N}\right) & 0\\ T_{21}\left(\frac{n(a+b)}{N}\right) & T_{22}\left(\frac{n(a+b)}{N}\right)\end{bmatrix}\begin{bmatrix}T_{11}\left(\frac{(n-1)(a+b)}{N}\right) & 0\\ T_{21}\left(\frac{(n-1)(a+b)}{N}\right) & T_{22}\left(\frac{(n-1)(a+b)}{N}\right)\end{bmatrix}\begin{pmatrix}A_{2(n-2)+1}\\B_{2(n-2)+1}\end{pmatrix}$$

$$=\begin{bmatrix}T_{11}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right) & 0\\ T_{21}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right)+T_{22}\left(\frac{n(a+b)}{N}\right)T_{21}\left(\frac{(n-1)(a+b)}{N}\right) & T_{22}\left(\frac{n(a+b)}{N}\right)T_{22}\left(\frac{(n-1)(a+b)}{N}\right)\end{bmatrix}\times\begin{pmatrix}A_{2(n-2)+1}\\B_{2(n-2)+1}\end{pmatrix}. \tag{15}$$

$B_{2(n-2)+1}$ is the imperfect trap coefficient that refers to the trap just before the discussed two-trap section and B_{2n+1} is the one that refers to the second trap in this specific section. $A_{2(n-2)+1}$ and A_{2n+1} are the corresponding ideal trap coefficients. The matrix equation (15) may be decomposed to yield the following expressions for the relevant coefficients:

$$A_{2n+1}=T_{11}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right)A_{2(n-2)+1}, \tag{16}$$

$$B_{2n+1}=\left[T_{21}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right)+T_{22}\left(\frac{n(a+b)}{N}\right)T_{21}\left(\frac{(n-1)(a+b)}{N}\right)\right]A_{2(n-2)+1}$$
$$+T_{22}\left(\frac{n(a+b)}{N}\right)T_{22}\left(\frac{(n-1)(a+b)}{N}\right)B_{2(n-2)+1}. \tag{17}$$

Using Eqs. (15) and (16) we may write Eq. (17) as

$$\frac{B_{2n+1}}{B_{2(n-2)+1}}=\left(\frac{T_{21}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right)+T_{22}\left(\frac{n(a+b)}{N}\right)T_{21}\left(\frac{(n-1)(a+b)}{N}\right)}{T_{11}\left(\frac{n(a+b)}{N}\right)T_{11}\left(\frac{(n-1)(a+b)}{N}\right)}\right)\frac{A_{2n+1}}{B_{2(n-2)+1}}$$
$$+T_{22}\left(\frac{n(a+b)}{N}\right)T_{22}\left(\frac{(n-1)(a+b)}{N}\right). \tag{18}$$

In order to be able to solve the last equation for $B_{2n+1}/B_{2(n-2)+1}$ we use, as done in Ref. [8], the assumption that the larger is the number of imperfect traps the classical particles pass the smaller becomes their ideal trap component compared to the imperfect one. In Ref. [8] which discuss, especially, the case of a very large number of traps the ratio of the ideal trap component at the last trap to the imperfect one at the first trap was equated to zero. We discuss here the case of a small number m so we may assume that this ratio depends on m and $B_{2n+1}/B_{2(n-m)+1}$ as

$$\frac{A_{2n+1}}{B_{2(n-m)+1}}=\frac{1}{(1+m^2)}\frac{B_{2n+1}}{B_{2(n-m)+1}}. \tag{19}$$

The last expression ensures that the ratio at the left-hand side vanishes in the limit of a very large m where $B_{2n+1}/B_{2(n-m)+1}$ tend to unity [8]. Denoting the expression that multiply $A_{2n+1}/B_{2(n-2)+1}$ in the first term on the right-hand side of Eq. (18) as $c_1^{(2)}$ and the second term as $c_2^{(2)}$ we write Eq. (18), using Eq. (19) in which $m=2$, as

$$\frac{B_{2n+1}}{B_{2(n-2)+1}}=\frac{c_2^{(2)}}{\left(1-\frac{c_1^{(2)}}{5}\right)}. \tag{20}$$

We may generalize the last equation that was written for the two-trap section for any finite number m of traps so that the corresponding analog of Eq. (20) is

$$\frac{B_{2n+1}}{B_{2(n-m)+1}}=\frac{c_2^{(m)}}{\left(1-\frac{c_1^{(m)}}{(1+m^2)}\right)}, \tag{21}$$

where it may be shown that $c_1^{(m)}$ and $c_2^{(m)}$ are given by the following recursive equations:

$$c_1^{(m)}=\frac{T_{21}\left(\frac{n(a+b)}{N}\right)+T_{22}\left(\frac{n(a+b)}{N}\right)c_1^{(m-1)}}{T_{11}\left(\frac{n(a+b)}{N}\right)}, \tag{22}$$

$$c_2^{(m)}=c_2^{(m-1)}T_{22}\left(\frac{n(a+b)}{N}\right). \tag{23}$$

Note that parameter m refers to the finite m trap system, which is a susbsystem of the N multitrap one, whereas parameter n denotes the general term of the last system (it actually refers to the position of the last trap of the subsystem in the larger N trap system). Now, it may be shown from the definitions of the variables $c_1^{(m)}$ and $c_2^{(m)}$ that the range of $c_2^{(m)}$ that involves the quantities T_{22} is in the interval (0,1) and that of $c_1^{(m)}$ that involves T_{11} and T_{21} is in $(-\infty, +\infty)$. Also, it may be seen that $c_1^{(m)}$ grows in absolute value with m and $c_2^{(m)}$ decreases to zero with increasing m so that in the limit of very large m the transmission amplitude tends to unity, as shown in Ref. [8]. The same result is obtained also for increasing c where $c_1^{(m)}$ decreases to zero and $c_2^{(m)}$ increases to its maximum value of unity so that in the limit of very large c the transmission amplitude tends to unity as may be seen from Eq. (21) (see also [8]).

IV. RESULTS FOR SOME GIVEN VALUES OF THE TRAP SYSTEM PARAMETERS *m*, *L*, *k*, AND *t*

Unlike the discussion in Ref. [8] which, especially, takes account of a large number of traps we discuss here, as remarked, the influence of a small number of them. First of all one finds, as expected, that the smaller is the number of traps m the easier is for the classical particles to pass through them. The criterion for this transmission is, as remarked (see [8]), the ratio of the value of the imperfect trap coefficient at the last trap (of the m member subsystem) to its value at the first one. This is the ratio $B_{2n+1}/B_{2(n-m)+1}$ from Eq. (21), which may be regarded as a transmission amplitude. Note that this amplitude may have values outside the range of $(0,1)$. An easy passage through the trap system is obtained not only for small values of m but also for large values of the total length L of the system as we have found in Ref. [8] for the multitrap case. The same result is obtained also for large k. Note that the nature of the change of the transmission amplitude with time is opposite to that regarding k and L. That is, this amplitude decreases with increasing time. Moreover, this decrease occurs in a very fast manner, especially at the initial time, compared to the remarked increase with k and L, as may be seen in Figs. 6–8.

Each of the following eight figures contains six curves of the transmission amplitude from Eq. (21) as functions of parameter c for the six values of $m=1,3,5,7,9,11$. As remarked and shown in Ref. [8] with respect to the multitrap system this transmission amplitude tends to unity for large values of c. We find here the dependence of the transmission upon c for small m and the small range of $0.001 \leqslant c \leqslant 20$. All the curves in the eight figures are drawn for the specific values of $D_e=0.5$ and $D_i=0.1$. These values yield results that are qualitatively similar for a wide class of different applications that use the trap system as a model [for example, 0.5 cm^2/s is the order of magnitude, one may find in the literature for the diffusion constant D at room temperature and atmospheric pressure (p. 337 in Ref. [13])]. We find that the larger is m the slower is the approach of its corresponding curve to unity compared to that of the smaller m curves. Thus, not all the six curves for $m=1,3,5,7,9,11$ are actually shown in each figure as in Figs. 1 and 4 in which the larger m curves are merged with the abcissa axis. The correct order of the curves in each figure is downward so that smaller values of m fit the upper curves (the graph for $m=1$ is the upper one, that for $m=3$ is the second from above and so on).

The group of Figs. 1–8 demonstrate this behavior of the transmission amplitude from Eq. (21) as function of c. The first three figures, each composed of six curves for $m=1,3,5,7,9,11$, show how the transmission amplitude changes with c for the same values of $k=1$ and $t=1$ but three different values of $L=5,13,27$. Figure 1 is drawn for $L=5$ and shows only the curves for $m=1,3,5,7$, whereas those for $m=9,11$ are merged with the abcissa axis. Figure 2, which is for $L=13$, shows all the six curves approaching unity as c grows but, as remarked, this approach is slower the larger is m. Figure 3, which is drawn for $L=27$, shows once again all the six curves approaching unity but now even the larger m-value curves tend to unity already at small values of c compared to Figs. 1 and 2. Thus, as remarked and as shown for the multitrap system in Ref. [8], the approach of the transmission amplitude to unity is more apparent and faster, even for small c, the larger is L.

Figures 4 and 5 show how the transmission amplitude as a function of c changes with k. Each curve from the total six curves of each figure in the group of Figs. 4 and 5 is drawn for $L=5$ and $t=1$. Figure 4 is for $k=5$ and one may see only the curves for the four smaller values of m that tend to

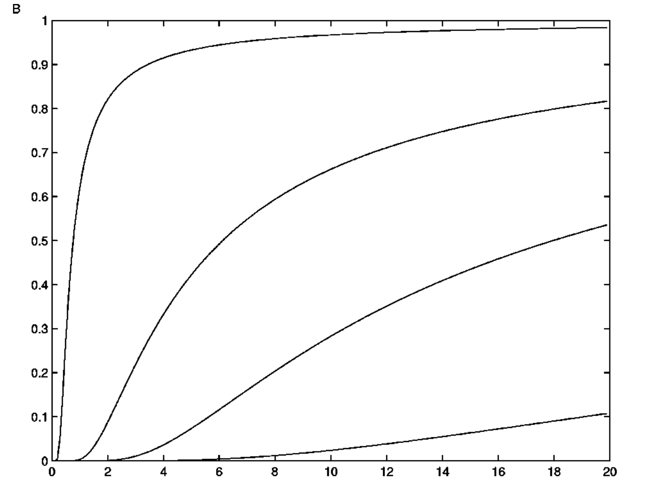

FIG. 1. The four curves show the transmission amplitude from Eq. (21) as functions of the ratio c for $m=1,3,5,7,9,11$. Note that the curves for the larger m values merge with the abcissa axis and are not shown. All the curves are drawn for the same values of $L=5$, $k=1$, $t=1$ and they all tend to unity for large values of c. As seen, the larger is m the slower is its approach to unity compared to that of the smaller m.

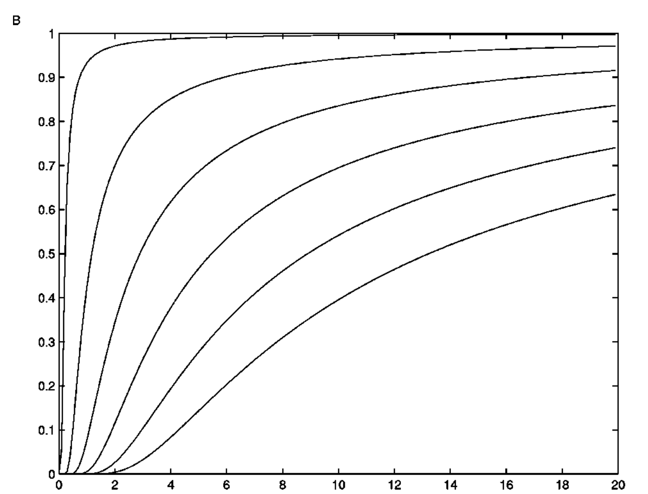

FIG. 2. The six curves show the transmission amplitude from Eq. (21) as functions of the ratio c for exactly the same values of m, k, and t as those of Fig. 1 but for $L=13$. Note that due to the larger L value all the six curves are shown (compare with Fig. 1). As in Fig. 1 (and all the other figures of this work) all the curves tend to unity for large c where the approach to unity is slower for the larger m values.

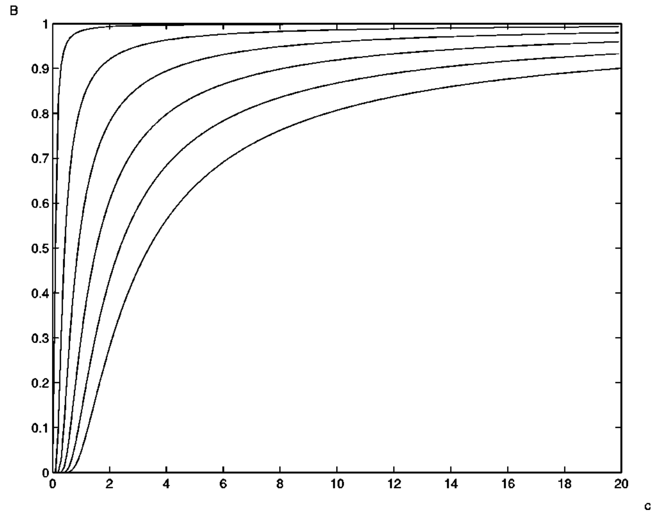

FIG. 3. The curves here are drawn under exactly the same conditions and for the same values of m, k, and t as those of Figs. 1 and 2 except that $L=27$. Comparing this figure to the former two figures one realizes that the curves approach unity not only for large c but also for large L.

unity for increasing c. The other two curves for $m=9,11$ are merged with the abcissa axis. Figure 5, which is drawn for $k=27$, shows now all the six curves approaching unity for increasing values of c. Thus, as remarked, the higher k values guarantees an easy transmission of the passing particles through the system. Note that, as remarked, this high transmission for increasing k is contrary to what one may expect that large k entails a large absorption [2,3,8] of the passing particles. The deviation of the obtained results from the expected ones is because the trap problem we try to solve here, as in Ref. [8], is the imperfect trap one and not the ideal one. Thus, the initial and boundary-value conditions employed are not the ideal ones [set (3)] but the imperfect [set (4)] and these may cause a large transmission even at the ideal trap limit of $k\rightarrow\infty$ as actually shown in Ref. [8] (see Fig. 2 there). The presence of the time factor in the initial and boundary-value conditions introduces interesting results that do not appear in the absence of it. For example, the analogous quantum one-dimensional multibarrier system along a finite section [11] does not involve any time variation and as a consequence the kind of change with time found here is not encountered there [11]. This kind of change is especially realized in the much more apparent and conspicuous manner, compared to that encountered for L and k, by which the transmission amplitude as a function of c changes for different values of time t. First of all, unlike the cases for k and L, this amplitude decreases with increasing t especially at the initial values of it. This is seen in Figs. 6–8 where all the six curves in each figure is drawn for $L=13$ and $k=7$. The curves of Fig. 6 are graphed for $t=0.01$ and one may see that all the six curves tend uniformly as a single graph to unity already at small values of c. Figure 7 is drawn for $t=1.6$ and one may see how at the small time span of 1.59 the curves become widely separated from each other so as those that correspond to the higher m values tend slowly to unity compared to those of the lower m. This form of the figure generally remains stabilized with time and changes only slightly by further increasing the time. In other words, a very large change in the behavior of the transmission amplitude, as a function of c, occurs during the initial time and then it remains almost stabilized. To further demonstrate the large influence of time we show in Fig. 8 the transmission amplitude, as function of c, for $L=13$, $k=7$, and $t=1$ and for $m=1,3,5,7,9,11$. A very similar figure is shown in Fig. 2, which is drawn for the same values of L, t, and m but for $k=1$; that is, increasing k from $k=1$ by 6 units, keeping the same values of $L=13$ and $t=1$, have a negligible influence

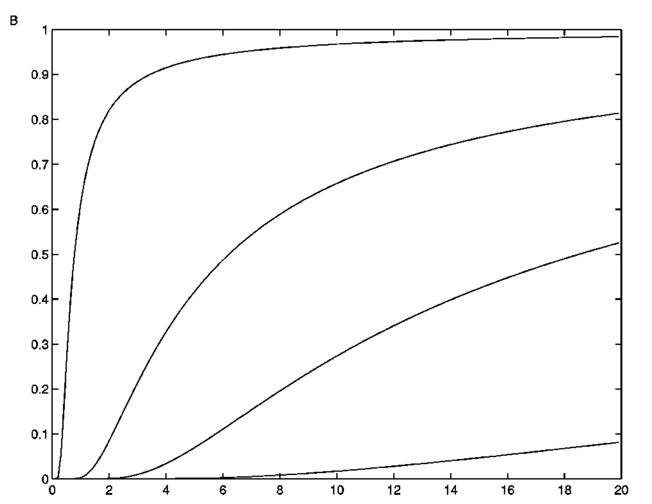

FIG. 4. The four curves show the transmission amplitude from Eq. (21) as functions of the ratio c for $m=1,3,5,7,9,11$. As in Fig. 1 the curves for the larger m values merge with the abcissa axis and are not shown. All the curves are drawn for the same values of $L=5$, $k=5$, and $t=1$ and they all tend to unity for large values of c where this approach is slower for larger m.

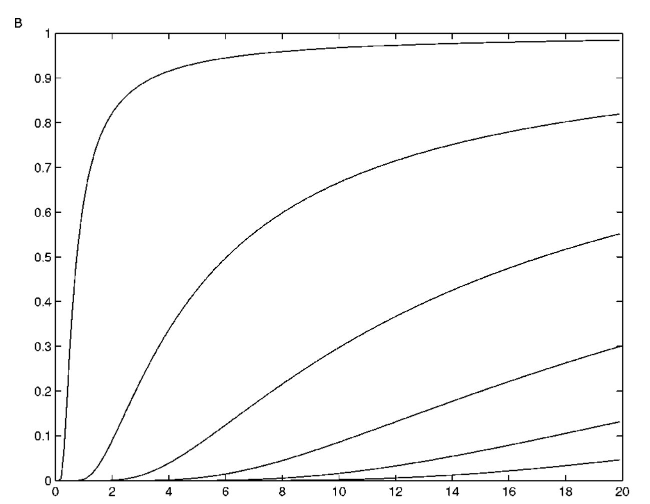

FIG. 5. The curves here are drawn under exactly the same conditions and for the same values of m, L, and t as those of Fig. 4 except that $k=27$. Note that due to the larger k value all the six curves are shown (compare with Fig. 4). Comparing this figure to the former four figures one realizes that the curves approach unity not only for large c and L but also for large k.

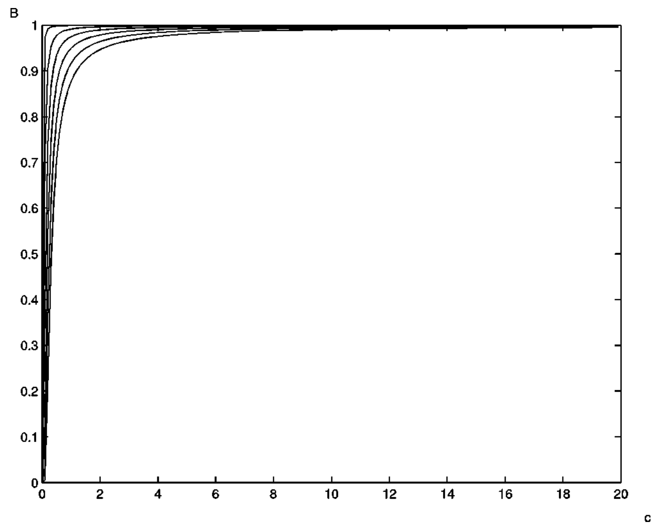

FIG. 6. The six curves show again the transmission amplitude from Eq. (21) as functions of c for the values of $m=1,3,5,7,9,11$, and for $L=13$, $k=7$, and $t=0.01$. Compared to the former figures one sees that for this small value of t all the curves approach almost immediately and together to unity. That is, for small t the six curves do not differ much from each other.

upon the transmission amplitude. But increasing the time by only 0.6, keeping the former values of L, m, and k, results in a discernable effect upon the transmission amplitude as shown in Fig. 7, which is drawn for the same values of L, k, and m, as in Fig. 8 but at $t=1.6$ (compare the two Figs. 7 and 8).

V. CONCLUDING REMARKS

We have discussed in this work the effects of a one-dimensional trap system upon the density of the passing classical particles. We have limit our discussion to the case of small number of traps (the large number case was discussed in Ref. [8]). As our analytical means we use the transfer matrix method discussed in Ref. [8] with respect to the one-dimensional multitrap system. We have shown that the transmission amplitude tends to unity, for growing c, not only in the limit of a very large number of traps as in Ref. [8] but also, under certain conditions, for the small number of them. These conditions involve either a large value of the parameter k or of the total length L of system. These results have been exemplified for specific values of k and L and demonstrated by the attached figures.

Unlike the remarked change of this amplitude with respect to k and L it has an opposite behavior regarding time t; that is, it decreases for all values of m as t increases where this decrease is larger for large m. Also, compared to k and L, this change with time is very fast, especially at the initial time and then the transmission amplitude stabilizes and changes only slightly with time. We have also shown for small m, as for the multitrap system in Ref. [8], that the imperfect character of the system, which is expressed in its initial and boundary-value conditions causes it to behave contrary to what is expected for large k. That is, although large value of k indicates, as remarked, a large absorption of the passing particles, nevertheless, we find a high transmission for large k due to the appearance of time in the initial and boundary-value conditions. The large influence of time upon the transmission amplitude have been shown and demonstrated in Figs. 6–8.

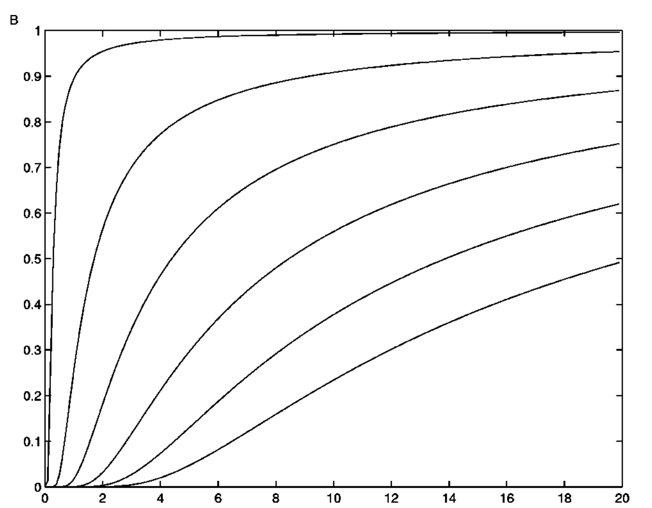

FIG. 7. The curves here are drawn under the same conditions and for the same values of m, L, and k as those of Fig. 6 except that $t=1.6$. Note the large change caused to the tramnsmission amplitude by slightly increasing the time (compare with Fig. 6). Also note that the character of the change with t is opposite to that with k and L, that is, the transmission amplitude decreases with increasing time.

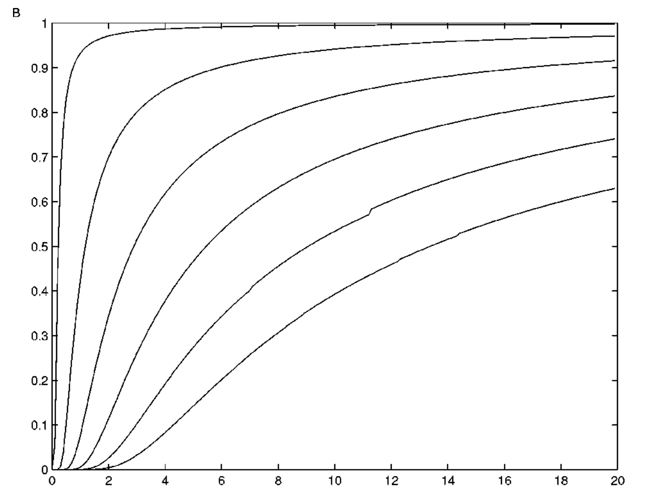

FIG. 8. The figure is drawn for the same values of L, k, and m as those of Fig. 7 but for $t=1$. Comparing Fig. 8 with Fig. 2, which was drawn for the same values of L and t but for $k=1$, one realizes that increasing k by 6 units, keeping the same values of L and t, has very little influence upon the transmission amplitude. But increasing the time from the value it has in this figure by only 0.6, keeping the same values of L and k, results in an apparent difference (compare Figs. 7 and 8).

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